# Written Exam at the Department of Economics summer 2018 

## Economic Growth

Final Exam

June 1, 2018

## (3-hour closed book exam)

Answers only in English.

This exam question consists of 5 pages in total (including this one)

NB: If you fall ill during an examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. In this connection, you must complete a form. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

## Be careful not to cheat at exams!

- You cheat at an exam, if during the exam, you:
- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam


## ECONOMIC GROWTH EXAM SPRING 2018

## Part 1: Essay questions

Question 1.a: Explain what the two (equally valid) conditions are for endogenous growth to arise in the one sector setting.

Question 1.b: What forces, related to the diffusion of ideas, may generate the prediction of conditional convergence? Conditional convergence also arises in standard Neoclassical theories of growth. Is it possible to distinguish the two theories empirically? If so, how?

Question 1.c: Please, describe the key differences between the Romer-model and the Aghion-Howitt model.

Question 1.d: What is misallocation and how does it affect aggregate output? Give two examples of misallocation and explain why such misallocations may arise.

Question 1.e: The gravity equation is used in the study by Frankel and Romer (1999). What empirical critique can be offered vis-a-vis their empirical work? How might the Suez crisis help us address this criticism?

Question 1.f: Figure 3 in Bloom et al (2018), reproduced here, shows that the number of transistors on an integrated circuit has doubled every two years since 1970. Why do Bloom et al (2018) nevertheless think research productivity has fallen both for transistors and elsewhere in the economy? What model(s) of economic growth is consistent with their evidence? What does their results imply for future economic growth?

## Part 2: Skill biased technical change in the task-based model

Consider the task-based model in Autor and Acemoglu (2011). Aggregate production is a Cobb-Douglas aggregate of a continuum of distinct tasks:

$$
Y=\exp \left[\int_{0}^{1} \ln y(i) d i\right]
$$

Each task is produced according to

$$
y(i)=A_{L} \alpha_{L}(i) l(i)+A_{M} \alpha_{M}(i) m(i)+A_{H} \alpha_{H}(i) h(i)
$$

Figure 3: The Steady Exponential Growth of Moore's Law

where $l(i), m(i)$ and $h(i)$ denote low-skilled labor, medium-skilled labor, and high skilled labor, respectively, employed in producing task $i . A_{L}, A_{M}, A_{H}$ are general skill-specific productivity levels, and $\alpha_{L}$, $\alpha_{M}$ and $\alpha_{H}$ are task and skill-specific productivity levels. Let $p(i)$ denote the price of task $i$. Assume that $\alpha_{L}(i) / \alpha_{M}(i)$ and $\alpha_{M}(i) / \alpha_{H}(i)$ are continuously differentiable and strictly decreasing such that low-skilled workers produce tasks $i<I_{L}$, high-skilled workers produce tasks $i>I_{H}$, and medium-skilled workers produce tasks $I_{L} \leq i \leq I_{H}$. Lastly, labor markets clear such that:

$$
\int_{0}^{1} l(i) d i=L, \quad \int_{0}^{1} m(i) d i=M, \quad \text { and } \quad \int_{0}^{1} h(i) d i=H
$$

Question 2a: Show that the equilibrium labor allocation is as follows:

$$
\begin{aligned}
l(i) & =\frac{L}{I_{L}} \text { for any } i<I_{L} \\
m(i) & =\frac{M}{I_{H}-I_{L}} \text { for any } L_{L}<i<I_{H} \\
h(i) & =\frac{H}{1-I_{H}} \text { for any } i>I_{H}
\end{aligned}
$$

Hint: You will need to derive the wage rates for each type of worker, and use the property of the symmetric Cobb-Douglas production function that expenditures on each task are identical.

Question 2b:Derive the two no-arbitage conditions for labor at $I_{L}$ and $I_{H}$ :

$$
\frac{A_{M} \alpha_{M}\left(I_{H}\right) M}{I_{H}-I_{L}}=\frac{A_{H} \alpha_{H}\left(I_{H}\right) H}{1-I_{H}}
$$

$$
\frac{A_{L} \alpha_{L}\left(I_{L}\right) L}{I_{L}}=\frac{A_{M} \alpha_{M}\left(I_{L}\right) M}{I_{H}-I_{L}}
$$

Question 2c: Use the nor-arbitrage conditions to illustrate in a ( $I_{H}, I_{L}$ ) diagram what happens to $I_{H}$ and $I_{L}$ if $A_{H}$ increases. Explain the intuition.
Question 2d: Based on the answer to the previous question, explain (intuitively) what happens to the relative wage rates $\frac{w_{H}}{w M}, \frac{w_{H}}{w_{L}}$ and $\frac{w_{M}}{w_{L}}$. Is the model consistent with what we observe in the data?

## Part 3: Endogenous growth

Consider an economy inhabited by infinitely lived agents. Time in continous. Along an optimal path, consumption follows the consumption euler: $\dot{c}_{t} / c_{t}=(1 / \theta)\left(r_{t}-\rho\right)$, where $c$ is consumption, $\theta$ is the elasticity of marginal utility, $r$ is the real rate of return whereas $\rho$ is the rate of time preference. The representative firm operates the production function

$$
Y_{t}=A L^{1-\alpha} \sum_{i=1}^{N_{t}} X_{i t}^{\alpha}
$$

where $A$ (total factor productivity) and $L$ (the labor force) are assumed constant. $N$ is the number of intermediate goods in existence at any given point in time, and $X$ is the use of each intermediate good $i=1, \ldots, N$. The price of the $\mathrm{i}^{\prime}$ th intermediate good is $p_{i}$ and that of labor is $w$. The price of output is normalized to 1 .

Question 3.a: (a) Comment on the production function. (b) Assuming the representative firm profit maximizes, derive the demand for intermediate good $i$.

Question 3.b: Intermediate goods are produced under Monopoly. That is, once an intermediate good has been invented the innovator can attain a patent of infinite duration. Assume one unit of $X$ can be produced using one unit of output. Profits, $\pi$, are thus given by $\pi_{i}=p_{i} X_{i}-X_{i}$. Show that the optimal price and quantity of the monopolist fullfills:

$$
\begin{gathered}
X_{i}=X=\alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L \text { for all } i \\
p_{i}=p=\frac{1}{\alpha} \text { for all } i
\end{gathered}
$$

Question 3.c: The value of a patent, $V_{t}$, fulfills $r_{t} V_{t}=\pi_{t}+\dot{V}_{t}$. Moreover, we assume that if an innovator invests $\eta$ units of output he or she recovers 1 idea deterministically. In equilibrium, therefore

$$
\eta=V_{t}=V \text { for all } t
$$

(a) Explain why the above condition should hold in equilibrium with positive $R \& D$ levels. (b) Show that in equilibrium:

$$
r=\frac{1}{\eta} \frac{\alpha}{1-\alpha} \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L
$$

Question 3.d: (a) Provide an argument that the present model does not involve tranditional dynamics and proceed to show that the growth rate of GDP in the economy is

$$
\gamma=\frac{1}{\theta}\left(\frac{1}{\eta} \frac{\alpha}{1-\alpha} \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L-\rho\right)
$$

(b) Is this growth rate likely to be socially optimal? Why/Why not?

